Numeracy for Cost Analysts
Doing the Right Math, Getting the Math Right

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Tim Anderson
Steve Book
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Nathan Menton
John Neatrour
Outline

- Motivation
- Definitions
- Why Numeracy is Critical to Cost Analysis
- What Right Looks Like
- Good Math, Bad Math
- Limitations of Math
- Summary
- The Last Word
- Acronyms
Motivation

• No one does costing right and it may never be possible to do so
• For now we must settle for doing it less wrong
• Proper mathematical and statistical techniques are necessary (but admittedly not sufficient) for doing costing less incorrectly
  – The use of incorrect techniques has been a contributor to chronic underestimation of cost and undermines confidence in the analysis
  – Math is one of the very few ingredients in an estimate for which there are objective criteria for goodness so when it is used, it needs to be done right
Obligatory Dilbert Cartoon

DO YOU HAVE THOSE BUDGET NUMBERS FROM LAST MONTH?

THEY'RE TOTALLY INACCURATE.

I KNOW, BUT THOSE ARE THE ONLY INACCURATE NUMBERS WE HAVE.

ACTUALLY, WE HAVE INFINITELY MANY NUMBERS TO CHOOSE FROM.

LET'S KEEP THOSE IN OUR BACK POCKET IN CASE WE NEED THEM.

I'LL ENCRYPT THEM SO NO ONE ELSE CAN USE THEM.

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“Numeracy is a contraction (or portmanteau word) of 'numerical literacy', and refers to an ability to reason with numbers and other mathematical concepts. The word was coined in 1959 by the UK Committee on Education, presided over by Sir Geoffrey Crowther...

“In the United States, numeracy it is also known as Quantitative Literacy, and is familiar to math educators and intellectuals. There is also substantial overlap between conceptions of numeracy and conceptions of statistical literacy.”
Main Entry: **numeracy**

**Pronunciation:** \'n(y)üm-rə-sē, 'n(y)ü-mə-

**Function:** *noun*  
**Etymology:** Latin *numerus* number + English *-acy* (as in *literacy*)

**Date:** 1959 :

the capacity for quantitative thought and expression

— **numerate** \'nūm-rət, 'nü-mə-; 'nyūm-, 'nyü-mə\- adjective
“Innumeracy” is a portmanteau of ‘numerical illiteracy’; it refers to a lack of ability to reason with numbers. The term innumeracy was coined by cognitive scientist Douglas Hofstadter and popularized by mathematician John Allen Paulos in his 1989 book, *Innumeracy: Mathematical Illiteracy and its Consequences*. Possible causes of innumeracy are poor teaching methods and standards and lack of value placed on mathematical skills. Even prominent and successful people will attest, sometimes proudly, to low mathematical competence, in sharp contrast to the stigma associated with illiteracy.”

“The problem with remaining innumerate is that one can be fooled not only by numerates but also by other innumerates.”.....NDH
Why Numeracy is Critical to Cost Analysis
A Cost Estimate Is a Probability Distribution, Not a Single Number

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>516.81</td>
</tr>
<tr>
<td>20%</td>
<td>538.98</td>
</tr>
<tr>
<td>30%</td>
<td>557.85</td>
</tr>
<tr>
<td>40%</td>
<td>575.48</td>
</tr>
<tr>
<td>50%</td>
<td>592.72</td>
</tr>
<tr>
<td>60%</td>
<td>609.70</td>
</tr>
<tr>
<td>70%</td>
<td>629.19</td>
</tr>
<tr>
<td>80%</td>
<td>650.97</td>
</tr>
<tr>
<td>90%</td>
<td>683.01</td>
</tr>
</tbody>
</table>

**Statistics**

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trials</td>
<td>10,000</td>
</tr>
<tr>
<td>Mean</td>
<td>596.40</td>
</tr>
<tr>
<td>Median</td>
<td>592.72</td>
</tr>
<tr>
<td>Mode</td>
<td>---</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>63.18</td>
</tr>
<tr>
<td>Range Minimum</td>
<td>450.19</td>
</tr>
<tr>
<td>Range Maximum</td>
<td>796.68</td>
</tr>
</tbody>
</table>

---

![Frequency Chart](image)

![Cumulative Chart](image)

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Some of the Math That Gets You to an Estimate

• Curve fitting methods to establish Cost Estimating Relationships (CERs)
  – Polynomial Fits
  – Regression techniques
    ▪ Ordinary Least Squares (OLS) and Log-OLS
    ▪ Iteratively Reweighted Least Squares (IRLS)
    ▪ Minimum Percent Error – Zero Percent Bias (MPE-ZPB)

• Statistical summing and other issues to get to the cost distributions
  – Monte Carlo and Latin Hypercube
  – Method of Moments
  – Correlation coefficients

• Spreading algorithms to get to annual funding
Math is the language of analysis and of cost analysis in particular!
What Right Looks Like
If We Could Only Emulate the Sciences....

1. Choose Your Topic

2. Identify a Problem

3. Research the Problem

4. Develop a Hypothesis

5. Design the Experiments

6. Test the Hypothesis

7. Analyze the Results

8. Formulate Conclusions
...to Develop the Best Math Model for Costing, it Would....

- Fit empirical data
- Be shaved by Occam’s Razor
- Be predictive across the broadest set of inputs
- Satisfy an appropriate theory and/or philosophical consideration
...But Cost Analysis is Not a Science...

• In 1605, Johannes Kepler completed his three laws of planetary motion
  – Derived from examination of data collected by Tycho Brahe
  – Phenomenologically, not theoretically based

• In 1687, Isaac Newton published his three laws of motion from which Kepler’s laws can be derived putting them on a causal basis

• Cost Analysis is in the phenomenological stage of development without a causal basis
  – CER development not tied to any theory
  – Methods are either statistical or heuristic
...So to the Greatest Extent Possible We Must

- Define terms clearly
- Apply theorems properly
  - Be sure to satisfy hypotheses before invoking the conclusions!
- Use appropriate mathematical formulations
  - The world is rarely linear
  - Let economics, engineering, and common sense be guides
- Do calculations correctly
  - Statistical summing is not done arithmetically
  - No, you may never average percents, ratios, or averages to create CERs, factors or models
BIT THE PRICE TO WIN

Subject: Management, marketing

The Rule

The bid price of an electro-optical system usually can be determined by the following relationship:

\[ \frac{P_1 + P_2 + P_3 - [\Delta P]}{S_b} = C \times \frac{B_1}{(D_o)^{1/3}} = Price \]

where:
- \( P_1 \): your competitor's price
- \( P_2 \): your cost price
- \( S_b \): mate bovine waste
- \( C \): methane gas
- \( B_1 \): crystal ball
- \( D_o \): management directive

Basis for the Rule

The rule is based on empirical observations, a cynical view of management theory, and the second law of thermodynamics.

Cautions and Useful Range of the Rule

It works for equally well for systems, component, and study contracts.

Be wary of your perceptions about your competitor's price; they are working as diligently as you to reduce costs.

This applies to price; the actual cost requires quite a different equation.

Usefulness of the Rule

This rule is useful for understanding some of the considerations that may go into your price or the price of your competitor.

Notes and Explanation

This is not an equation of equal; electro-optics is sufficiently high-tech that the cost is a strong function of "electro." If desired, most systems and components can have costly qualifications, iterations, testing, and features. These may or may not be required for basic operation. Hence, a large part of the price often is based on the (perceived) actions of the competitor, capricious decisions from management, and a tail of bovine waste. Observe that the management directive is only to the 1/2 power. This reflects the fact that in non-technical company, a management decision can be changed when faced with overwhelming technical evidence.

Source

Rule provided by Grant Milheiser, 1995.
Good Math, Bad Math
Telling Good from Bad

Appropriate Math Model

<table>
<thead>
<tr>
<th>Bad Math</th>
<th>Good Math</th>
</tr>
</thead>
</table>

Inappropriate Math Model

<table>
<thead>
<tr>
<th>Very Bad Math*</th>
<th>Bad Math</th>
</tr>
</thead>
</table>

Math Done Wrong | Math Done Right

* AKA Junk Mathematics

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Models Misused

• Ignoring Phase of the Program
  – Example: COCOMO II has two versions, Pre Design and Post Architecture

• Not fully utilizing all inputs when available

• Failing to use a range for inputs in favor of a single value

• Setting ranges for estimating error without regard to statistics that are associated with the development of the CERs
## Very Bad Math: A Contrived Example

So on average, contractors represent

\[ \frac{90\% + 40\% + 80\%}{3} = 70\% \text{ of the total O&S Staff} \]
First, the Average Must Be Weighted!

In general:

\[
\frac{1}{n} \sum_{i=1}^{n} \frac{a_i}{b_i} \neq \frac{\sum_{i=1}^{n} a_i}{\sum_{i=1}^{n} b_i} = \frac{\sum_{i=1}^{n} b_i \left( \frac{a_i}{b_i} \right)}{\sum_{i=1}^{n} b_i}
\]

In our example, the weighted average is:

Total Number of Contractors \[\frac{9 + 200 + 40}{10 + 500 + 50} = \frac{249}{560} = 44\%\]
Second, a Linear Relationship is Not Appropriate

Number of Contractor vs Total O&S Staff

\[ y = 1.601x^{0.7863} \]

\[ R^2 = 0.9936 \]
<table>
<thead>
<tr>
<th>Total Number of O&amp;S Staff</th>
<th>Actual Number of Contractor O&amp;S Staff</th>
<th>Estimated Number of Contractor O&amp;S Staff Using Percent</th>
<th>Delta (Actual - Estimated Using Percent)</th>
<th>Estimated Number of Contractor O&amp;S Staff Using Formula</th>
<th>Delta (Actual - Estimated Using Formula)</th>
<th>Delta (Estimated By Percent - Estimated Using Formula)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>9</td>
<td>4</td>
<td>5</td>
<td>10</td>
<td>-1</td>
<td>-5</td>
</tr>
<tr>
<td>50</td>
<td>40</td>
<td>22</td>
<td>18</td>
<td>35</td>
<td>5</td>
<td>-12</td>
</tr>
<tr>
<td>500</td>
<td>200</td>
<td>222</td>
<td>-22</td>
<td>212</td>
<td>-12</td>
<td>10</td>
</tr>
</tbody>
</table>
Very Bad Math: Real Example #1

- 21 total programs
  - Mix of government programs
  - 100% complete (two exceptions)
- Final contract values from $40M to $4,000M
- Average contract growth 78%

![Graph showing total growth over the years](image-url)
Real Example #1: Percents Averaged

<table>
<thead>
<tr>
<th>Program</th>
<th>Total Growth</th>
<th>Incidental ECPs</th>
<th>Technical ECPs</th>
<th>Cost Variance</th>
<th>Schedule Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Explicit</td>
<td>Implicit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>103%</td>
<td>0%</td>
<td>8%</td>
<td>9%</td>
<td>57%</td>
</tr>
<tr>
<td>2</td>
<td>119%</td>
<td>7%</td>
<td>5%</td>
<td>7%</td>
<td>100%</td>
</tr>
<tr>
<td>3</td>
<td>89%</td>
<td>13%</td>
<td>12%</td>
<td>16%</td>
<td>25%</td>
</tr>
<tr>
<td>4</td>
<td>54%</td>
<td>0%</td>
<td>10%</td>
<td>19%</td>
<td>26%</td>
</tr>
<tr>
<td>5</td>
<td>39%</td>
<td>4%</td>
<td>1%</td>
<td>8%</td>
<td>26%</td>
</tr>
<tr>
<td>6</td>
<td>10%</td>
<td>1%</td>
<td>6%</td>
<td>3%</td>
<td>FP</td>
</tr>
<tr>
<td>7</td>
<td>21%</td>
<td>0%</td>
<td>1%</td>
<td>17%</td>
<td>3%</td>
</tr>
<tr>
<td>8</td>
<td>15%</td>
<td>0%</td>
<td>12%</td>
<td>2%</td>
<td>0%</td>
</tr>
<tr>
<td>9</td>
<td>24%</td>
<td>1%</td>
<td>23%</td>
<td>0%</td>
<td>-1%</td>
</tr>
<tr>
<td>10</td>
<td>92%</td>
<td>0%</td>
<td>32%</td>
<td>30%</td>
<td>26%</td>
</tr>
<tr>
<td>11</td>
<td>53%</td>
<td>0%</td>
<td>20%</td>
<td>30%</td>
<td>1%</td>
</tr>
<tr>
<td>12</td>
<td>13%</td>
<td>0%</td>
<td>11%</td>
<td>-1%</td>
<td>3%</td>
</tr>
<tr>
<td>13</td>
<td>80%</td>
<td>0%</td>
<td>5%</td>
<td>7%</td>
<td>53%</td>
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<td>5%</td>
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<td>1%</td>
<td>6%</td>
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<tr>
<td>16</td>
<td>235%</td>
<td>0%</td>
<td>16%</td>
<td>11%</td>
<td>206%</td>
</tr>
<tr>
<td>17</td>
<td>240%</td>
<td>0%</td>
<td>25%</td>
<td>4%</td>
<td>103%</td>
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<td>18</td>
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<td>18%</td>
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<td>19</td>
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<td>13%</td>
<td>7%</td>
<td>0%</td>
</tr>
<tr>
<td>20</td>
<td>84%</td>
<td>0%</td>
<td>0%</td>
<td>2%</td>
<td>82%</td>
</tr>
<tr>
<td>21</td>
<td>85%</td>
<td>1%</td>
<td>3%</td>
<td>11%</td>
<td>45%</td>
</tr>
</tbody>
</table>

Average: 78% 2% 12% 10% 43% 11%

This is the sum of the five averages to the right!

These five averages are computed without weighting.
**Very Bad Math: Real Example #2**

<table>
<thead>
<tr>
<th>Program Name</th>
<th>Subsystem</th>
<th>Non-Recurring</th>
<th>First Unit (T1)</th>
<th>NR/T1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Program 1</td>
<td>Subsystem X</td>
<td>13.6</td>
<td>17.9</td>
<td>0.76</td>
</tr>
<tr>
<td>Program 2</td>
<td>Subsystem X</td>
<td>0.8</td>
<td>0.6</td>
<td>1.26</td>
</tr>
<tr>
<td>Program 3</td>
<td>Subsystem X</td>
<td>6.5</td>
<td>6.2</td>
<td>1.05</td>
</tr>
<tr>
<td>Program 4</td>
<td>Subsystem X</td>
<td>3.7</td>
<td>2.1</td>
<td>1.78</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td></td>
<td><strong>24.6</strong></td>
<td><strong>26.8</strong></td>
<td></td>
</tr>
</tbody>
</table>

Method 1: Weighted Average = 24.6 / 26.8 = 0.92
Method 2: Simple Average = (0.76 + 1.26 + 1.05 + 1.78 / 4) = 1.21

- Use Weighted Average method to more heavily weight the larger systems
- Use the Simple Average method to weight each data point equally
- If Method 1 is used the NR Bus estimate would be reduced by $78M (BY03)
- Method 2 substantiated and recommended by our Statistician and Scientist
What’s Hot and What’s Not Among the Numerate

<table>
<thead>
<tr>
<th>Out</th>
<th>In</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Point Estimate”</td>
<td>Estimate with associated confidence level</td>
<td>“Point Estimate” is an undefined term. A proper cost estimate must reflect its probabilistic nature.</td>
</tr>
<tr>
<td>Arithmetic Summing of Estimates</td>
<td>Statistical Summing of Estimates</td>
<td>Only means of distributions can be arithmetically summed and nothing else!</td>
</tr>
<tr>
<td>OLS and Log-OLS</td>
<td>MPE-ZPB</td>
<td>The world is rarely linear and the assumptions for OLS are even more rarely satisfied. Multiplicative error makes more sense for cost estimates.</td>
</tr>
<tr>
<td>IRLS = Minimum Unbiased Percentage Error (MUPE)</td>
<td>IRLS ≠ MUPE</td>
<td>IRLS is biased in small samples (Goldberg and Sperling)</td>
</tr>
<tr>
<td>Learning Curves</td>
<td>Quantity as an Independent Variable (QAIV)</td>
<td>Learning slope is a significant cost driver but its selection rarely has a solid justification. Let the data dictate adjustment.</td>
</tr>
<tr>
<td>Black Box Cost Models</td>
<td>Transparent Cost Models</td>
<td>Should be obvious why.</td>
</tr>
<tr>
<td>Cost as an Independent Variable (CAIV)</td>
<td>Design to Cost (DTC)</td>
<td>Cost is a dependent random variable, not independent. The older terminology was better.</td>
</tr>
<tr>
<td>Innumeracy</td>
<td>Numeracy</td>
<td>‘Nuff said</td>
</tr>
</tbody>
</table>
The Limitations of Math
Well for One Thing, Math Won’t Make You Any Friends Among the Innumerate
Right Math Can Be Done Right But Still Yield A Counterintuitive Result

Average Unit Cost (FY07$K) of missile = 
\[ a + b(LQ^c)(1+PLQ)^d(MV^e)(f^{\text{FirstLot}}) \]

Where:
LQ is lot size, PLQ is previous lot quantity
MV is maximum velocity (m/s), FirstLot = 0 if Yes and 1 if No

and

\[ \begin{align*}
    a &= -0.62 \\
    b &= 0.00003625 \quad \text{Very small factor; gives more weight to other fit parameters} \\
    c &= -1.199 \quad \text{Large negative value causes total cost to decrease when lot size is increased} \\
    d &= -0.002 \\
    e &= 2.747 \quad \text{Very large value causes cost to be nearly cubed for each additional m/s of maximum velocity} \\
    f &= 0.612 \quad \text{Value less than one causes the first lot cost to be less than subsequent lots (opposite of learning curve theory)}
\end{align*} \]

Standard Percent Error = 52.7%
Percent Bias = 0.0%
\( R^2 = 52.0\% \)
Good Math Doesn’t Guarantee that

• Programs won’t overrun
  – Risk impacts may not be adequately considered
  – Budgets may not include a reasonable reserve

• Analyses will be accepted
  – Clients glaze over when math is shown
  – Clients often are inadequately trained in math and statistics
  – More attractive results (i.e., less scary) from less numerate costers prevail
The Inadequacy of Models

- Costs of programs are determined by more than just labor and material
- Drivers include management, political factors, and budgetary environment which don’t readily lend themselves to quantification
- We still lack an overarching theory that would yield better cost models
  - No guarantee that one exists
  - Investigation into similar disciplines such as economics and finance is yielding some ideas that may point to a causal foundation for cost analysis
Other Challenges

- Small data sets
- Assessing estimating errors
- Selection of figures of merit for models
- Determining most appropriate distributions for modeling impact of risk
- Creating joint cost and schedule probability distributions
Summary

• In life and particularly in cost analysis, innumeracy isn’t cool

• As cost analysts introduce more mathematical models and methods, care must be taken to follow the rules
  – Define terms
  – Satisfy hypotheses of theorems before applying them
  – Adopt the most appropriate model formulations
  – Do the calculations correctly

• Failure to do the right math and to get the math right will continue to introduce unnecessary error into our estimates and erode confidence in our numerate clients who know what good is
“You are entitled to your own opinion but not to your own mathematics”...NDH
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>BY</td>
<td>Base Year</td>
</tr>
<tr>
<td>CAIV</td>
<td>Cost as an Independent Variable</td>
</tr>
<tr>
<td>CER</td>
<td>Cost Estimating Relationship</td>
</tr>
<tr>
<td>COCOMO</td>
<td>Constructive Cost Model</td>
</tr>
<tr>
<td>DTC</td>
<td>Design to Cost</td>
</tr>
<tr>
<td>ECP</td>
<td>Engineering Change Proposal</td>
</tr>
<tr>
<td>ICE</td>
<td>Independent Cost Estimate</td>
</tr>
<tr>
<td>IRLS</td>
<td>Iteratively Reweighted Least Squares</td>
</tr>
<tr>
<td>MPE</td>
<td>Minimum Percent Error</td>
</tr>
<tr>
<td>MUPE</td>
<td>Minimum Unbiased Percentage Error</td>
</tr>
<tr>
<td>NDH</td>
<td>Neal David Hulkower</td>
</tr>
<tr>
<td>NR</td>
<td>Nonrecurring</td>
</tr>
<tr>
<td>NRO</td>
<td>National Reconnaissance Office</td>
</tr>
<tr>
<td>O&amp;S</td>
<td>Operations and Support</td>
</tr>
<tr>
<td>OLS</td>
<td>Ordinary Least Squares</td>
</tr>
<tr>
<td>Ph.D.</td>
<td>Piled Higher and Deeper</td>
</tr>
<tr>
<td>POE</td>
<td>Program Office Estimate</td>
</tr>
<tr>
<td>QAIV</td>
<td>Quantity as an Independent Variable</td>
</tr>
<tr>
<td>T1</td>
<td>Theoretical First Unit Cost</td>
</tr>
<tr>
<td>WBS</td>
<td>Work Breakdown Structure</td>
</tr>
<tr>
<td>ZPB</td>
<td>Zero Percent Bias</td>
</tr>
</tbody>
</table>