






MCR

Critical Thinking. Solutions Delivered.

Decisions, Decisions
Why You Oughta Use Borda

presented by:

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Acknowledgements

The author is grateful to the following for their review and suggestions:

Ray Covert

Jason Dechoretz

John Driessnack

John Neatrour

Sarah Springer

Outline

- Motivation
- Voting Methods
- Using Borda for Trades
- Summary: Why You Really Oughta Use Borda
- References
- Acronyms

Motivation (1 of 2)

- A primary responsibility of folks in our business is either to make decisions or to support those who do, e.g., form consensus
  - For example, cost analysts regularly contribute to trade studies of various types
- There are many important considerations in preparing to perform a trade study such as
  - Setting objective and mutually exclusive definitions of alternatives
  - Establishing assessment criteria
  - Selecting a mathematically sound decision method
- Let's look carefully at the latter

Motivation (2 of 2)

- Rigorous mathematics has established the unique superiority of the Borda Count among all positional voting schemes
- Traditional decision methods such as pairwise (e.g., Analytical Hierarchy Process (AHP)) or partwise comparisons and simple majorities continue to be used despite being resoundingly discredited (Saari and Sieberg - 2004)
- This presentation is intended to encourage adaptation of the Borda Count as the most defensible method of making decisions among all positional voting methods and any involving pairwise and partwise comparisons

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Voting Methods
Why the Choice is Important

"....the winner of an election may more accurately reflect the choice of a decision procedure rather than the views or preferences of the voters." **(Saari - 2001b, p.13)**

Positional Voting Methods

- Voters rank order candidates in order of preference with ties permitted
- Points are assigned based on rank with the candidate having the largest total points being declared the winner
- For three candidates, the notation for normalized points assigned in descending order of rank is (1,s,0) where $0 \le s \le 1$
  - Plurality voting, s = 0
  - Antiplurality voting, s = 1
  - The Borda Count, s = ½ (more on this method later)
  - Weighting method that disproportionately weights the points assigned; for example (5,2,1), is equivalent to (4,1,0) which normalizes to (1,¼,0) so s = ¼
- All possible positional voting methods lie along the line segment with plurality voting and antiplurality voting at opposite ends:
  $(1-s)(1,0,0) + s(1,1,0) = (1,s,0)$ where $0 \le s \le 1$
- For *n* candidates, notation for normalized points generalizes to $(1,s_1,s_2,\ldots,s_{n-2},0)$ where $0 \le s_{n-2} \le \ldots \le s_2 \le s_1 \le 1$

Different Winners

- Thirteen people want to select a party beverage from among Milk (M), Wine (W), and Beer (B)*. The list of voters' rankings, or profile, is as follows, where A > B means A is preferred to B

Number of voters	Ranking
4	M > W > B
2	W > B > M
1	B > M > W
2	M > B > W
4	B > W > M

- Milk is the Plurality winner, (1,0,0): M > B > W with tally 6:5:2
- Wine is the Antiplurality winner, (1,1,0): W > B > M with tally 10:9:7
- Beer is the Borda winner, (1,½,0): B > M > W with tally 7:6½:6

*Example from (Saari - 2001b,Subsection 1.3.1)

Other Examples of Voting Methods

- Pairwise comparisons (Condorcet method)
  - Candidates are compared in pairs in a majority vote
  - The Condorcet winner is the candidate that beats all others
- Approval voting: voters may vote for some or all of the candidates with the winner determined by the largest number of votes
- Run-offs
- Instant Run-offs
- Hybrids

A Bit About Borda

- Proposed by Jean-Charles de Borda in the late 18th century (Borda - 1781)
- Positional voting scheme
  - Each voter rank orders the candidates in order of preference with ties permitted
  - For *n* candidates, assigns (*n*-1) points to highest ranked candidate, (*n*-2) to second highest,...., 0 to the candidate ranked last.* Candidates that are tied each get the average of the total of the points for the positions they occupy
  - Borda winner is the candidate with the largest point total

*** The actual number of points assigned can vary and the order can be descending or ascending as long as the difference in points assigned neighboring candidates is constant.**

Four Rational Criteria

- Transitive outcomes: if A > B and B > C then A > C
- Pareto condition: if all voters prefer A to B then the outcome of the method also has A > B
- Unrestricted domain: allows each voter to select any strictly transitive ranking of alternatives
- Independence of Irrelevant Alternatives (IIA) requires that the outcome depends only on the relative ranking of any particular pair

Trouble is.....

Arrow's Theorem

- The only procedure that satisfies all four rational criteria on the previous slide is one where the outcome always agrees with the dictator's (or equivalently, one particular voter's) preference regardless of the preferences of other voters.
- In other words, "there is a specified voter so that the societal outcome always agrees with that voter's preference" (Saari - 2008, p.22)

Disposing Dictators

- Saari
  - recognized that IIA negates transitivity!
  - introduced a modification to IIA called Intensity of Binary Independence (IBI), also called the Intensity form of Independence of Irrelevant Alternatives (IIIA) , which has the overall ranking of any two alternatives determined by each voter's relative ranking and the intensity of that ranking
    - In other words, it matters not only if A > B but also if A > B > C > D or A > C > D > B
    - Looking only at the relative position of A and B and not considering the intensity ignores critical information

Borda is Better

- While Borda does not satisfy IIA, it does comply with IBI (IIIA) with the intensity of a ranking measured by the number of candidates between any two in consideration
- Because it uses more information than IIA, IBI does not negate transitivity of individual preferences thus significantly reducing the cause of unexpected outcomes
- By substituting IBI for IIA, Saari proved that the Borda Count is the only nondictatorial positional voting method that also satisfies Unrestricted Domain, Pareto and transitivity

References (Saari - 2000a, 2000b, 2001a, 2001b, 2008)

Unexpected Outcomes or Paradoxes: An Example*

- Consider the following profile: 10 voters rank A > D > C > B; another 10 rank B > A > D > C and a third set of 10 voters rank C > B > A > D
- Note that all 30 voters rank A>D
- The voting method used is series of three pairwise votes using the profile above
  - First pits B against A with B winning 20 to 10
  - Second between B and C, has C prevailing by 20 to 10
  - Finally, C falls to D by a vote of 20 to 10
- D is the winner even though the voters unanimously prefer A to D!
- This particular method, which plainly violates the Pareto condition, leads to an outcome that none of the voters would expect

*From (Saari - 2001b, Subsection 1.3.3)

A Major Cause of Paradoxes

- Condorcet n-tuples are rankings that end up in a complete tie
  - Condorcet 4-tuple is A > B > C > D, B > C > D > A, C > D > A > B, and D > A > B > C
  - Dropping D leaves the Condorcet triple A > B > C, B > C > A, C > A > B and also A > B > C, the source of inconsistencies
- Saari demonstrated that the Borda Count is unique among all positional voting methods in that “all possible inconsistencies in Borda rankings over subsets of candidates are strictly due to Condorcet terms” (Saari - 2008, p.157)
- Inconsistencies experienced by all other positional voting schemes are not only due to the Condorcet term but can also result from other causes

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Types of Trade Studies

	Analysis of Alternatives (AoA)	Economic Analysis (EA)	Business Case Analysis (BCA)
Decision Basis	• Cost • Capability Value of the System Solution	• NPV (Required) + non-monetary benefits • ROI / IRR / Payback Period (Secondary) Value to the Unit	• Alignment with organizational strategy • NPV / IRR / ROI Value to Organization
Scope/ Focus	• Analytical focus (operational effectiveness vs cost) • Identifies minimum set of alternatives	• Monetary focus • Limited scope (in practice)	• Enterprise/Strategic focus • Broad scope – considers far reaching implications (e.g. Industrial Base considerations)
Threshold	• ACAT I MS decisions • ACAT II and III (as directed)	• Initial investment > $1M • Rec. investment > $250K	• TBD (Threshold criteria might include operational, policy & financial consequences)

Chart was provided by Jason Dechoretz

Each Criterion Acts as a Voter

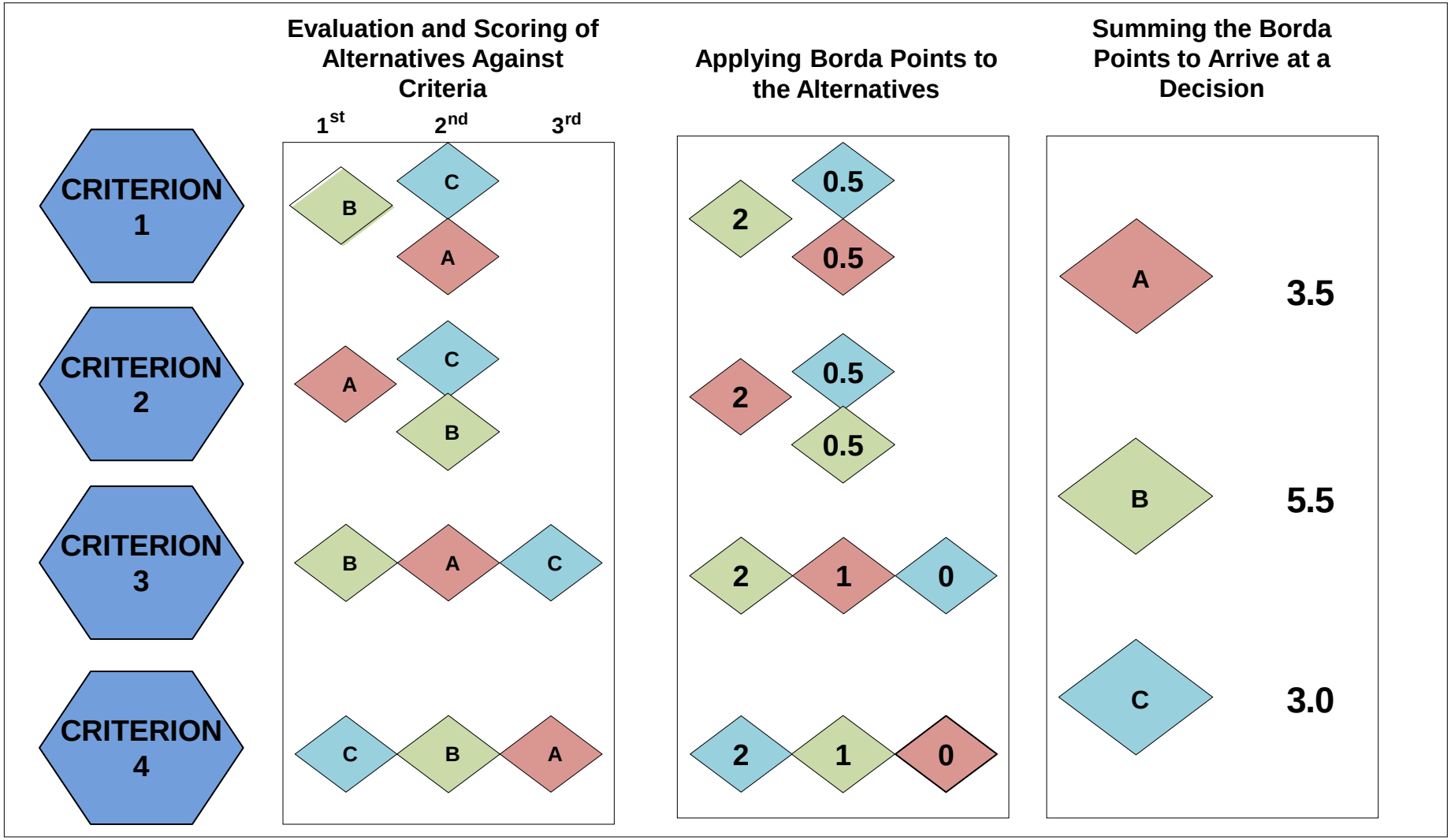

Illustration was provided by Andy Wehrle

Determining the Probable Lowest-Cost Alternative

- Recommended approach: Exact Probabilities by Simulation with Borda (Hulkower - 2010):
  - Determine the cost distributions for each candidate
  - Using Monte Carlo or Latin Hypercube sampling, calculate sufficiently large number (at least 5,000) of samples for each cost distribution
  - At each iteration, rank order in ascending order the cost drawn for each alternative with the lowest cost ranked highest and the highest cost, lowest
  - Use the Borda Count to determine the probable lowest cost alternative
- This method is an extension of Book's that only keeps track of the lowest cost alternative at each step of the iteration and selects as the probable lowest cost alternative the one that comes in lowest most frequently (Book - 2010)
  - This is equivalent to using the plurality method
  - By ranking the cost of each alternative, the outcome can differ

Example: The Distributions

Candidate	A	B	C	D
Mean	95	85	100	90
Standard Deviation	50	5	30	70

The Four Lognormal Cost Distributions

Note: These are the same distributions used in (Book - 2010)

Which of these would you expect to be the probable lowest-cost candidate? How would the others rank?

Example: The Outcome

Trial	COST (A)	COST (B)	COST (C)	COST (D)		A Rank	A Borda Score	B Rank	B Borda Score	C Rank	C Borda Score	D Rank	D Borda Score
1	214.16	83.15	86.20	114.68		4	0	1	3	2	2	3	1
2	135.94	87.67	105.34	114.41		4	0	1	3	2	2	3	1
3	101.53	83.36	85.13	58.04		4	0	2	2	3	1	1	3
4	114.80	79.87	113.79	104.86		4	0	1	3	3	1	2	2
5	52.68	84.32	50.09	61.10		2	2	4	0	1	3	3	1
6	122.70	78.71	128.86	220.20		2	2	1	3	3	1	4	0
7	129.13	78.10	49.62	74.52		4	0	3	1	1	3	2	2
8	105.91	84.91	82.64	57.66		4	0	3	1	2	2	1	3
9	89.89	81.96	66.38	56.16		4	0	3	1	2	2	1	3
10	54.40	81.09	83.76	20.76		2	2	3	1	4	0	1	3
...	...	...	...	...		...	...	...	...	...	...	...	...
...	...	...	...	...		...	...	...	...	...	...	...	...
...	...	...	...	...		...	...	...	...	...	...	...	...
9990	76.25	88.98	168.26	67.33		2	2	3	1	4	0	1	3
9990	78.44	76.21	147.71	17.07		3	1	2	2	4	0	1	3
9991	68.31	91.44	132.34	70.03		1	3	3	1	4	0	2	2
9992	98.67	90.27	55.72	122.22		3	1	2	2	1	3	4	0
9993	49.94	86.12	62.48	35.91		2	2	4	0	3	1	1	3
9994	33.52	92.24	92.91	32.62		2	2	3	1	4	0	1	3
9995	72.86	87.39	103.97	41.78		2	2	3	1	4	0	1	3
9996	132.37	84.02	163.71	43.62		3	1	2	2	4	0	1	3
9997	72.87	83.28	97.51	41.03		2	2	3	1	4	0	1	3
9998	68.87	87.07	85.23	264.63		1	3	3	1	2	2	4	0
9999	80.72	82.47	94.94	18.94		2	2	3	1	4	0	1	3
10000	91.86	89.36	90.98	107.13		3	1	1	3	2	2	4	0
						Sums =	15,188		15,486		10,989		18,337

Abbreviated Table of Simulation Outputs with Borda Scores

(Hulkower - 2010)

Example: The Conclusion

- Exact Probability by Simulation with Borda yields an aggregated ranking of D > B > A > C
- Exact Probability by Simulation (with Plurality Voting) yields an aggregated ranking of D > A > B > C
- Since Exact Probability by Simulation with Borda uses the complete information available at each step of the simulation to rank the alternatives and aggregates the rankings using the best positional voting scheme, its result in a more credible and defensible outcome
- The aggregated ranking of the alternatives from probable lowest-cost to probable highest-cost can be incorporated with the rankings by other criteria using the Borda Count to determine the most desirable alternative

(Hulkower - 2010)

Turning Pairwise Comparisons into the Borda Count

- In practice, it may be difficult to rank order a large number of alternatives especially when using subjective criteria
- In such circumstances, one may do pairwise comparisons keeping track of the results for each possible pair then aggregating them as shown in the example on the next chart to obtain the equivalent of the Borda Count (Saari and Sieberg - 2004)
  - The key is to keep track not only which alternative wins in each pairwise comparison but also the scores each alternative gets
  - These scores are summed across all pairwise outcomes for each alternative with the one getting the highest score declared the Borda winner

Turning Pairwise Comparisons into the Borda Count: An Example*

- Consider the profile:

5	A > B > C	4	A > C > B	4	C > B > A	6	B > A > C

- Using Borda, the outcome is A > B > C with A having 24 points, B having 21 point, and C having 12 points
- Now consider all pairwise contests:
  - A vs B: A wins (5+4=) 9 times and B wins (4+6=) 10 times
  - A vs C: A wins 15 times and C wins 4 times
  - B vs C: B wins 11 times and C wins 8 times
  - So A wins 9+15 = 24 times, B wins 10+11 = 21 times and C wins 4+8 = 12 times, reproducing the Borda totals and hence the outcome above

*From (Saari and Sieberg - 2004)

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Summary: Why You Really Oughta Use Borda

- Like many positional voting schemes but none of the partwise methods, the Borda Count uses all of the information provided in each voter's ranking to determine the overall outcome
- Also, it is the only nondictatorial positional voting method that has
  - Transitive outcomes and satisfies Unrestricted Domain, IIIA, and the Pareto Condition
  - Condorcet n-tuples as the sole cause of unexpected outcomes
- Implementation is straightforward, transparent, and requires no special software (Lansdowne and Woodward - 1996)

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Acronyms

- AHP Analytic Hierarchy Process
- AoA Analysis of Alternatives
- BCA Business Case Analysis
- BY Base Year
- EA Economic Analysis
- IBI Intensity of Binary Independence
- IIA Independence of Irrelevant Alternatives
- IIIA Intensity form of IIA
- IRR Internal Rate of Return
- NPV Net Present Value
- Ph.D. Piled Higher and Deeper
- ROI Return on Investment